Persistent spin and charge currents and magnification effects in open ring conductors subject to Rashba coupling

R. Citro and F. Romeo
Dipartimento di Fisica "E. R. Caianiello", Università degli Studi di Salerno, and Unità C.N.I.S.M., Via S. Allende, I-84081 Baronissi (SA), Italy
(Dated: February 5, 2008)

We analyze the effect of Rashba spin-orbit coupling and of a local tunnel barrier on the persistent spin and charge currents in a one-dimensional conducting Aharonov-Bohm (AB) ring symmetrically coupled to two leads. First, as an important consequence of the spin-splitting, it is found that a persistent spin current can be induced which is not simply proportional to the charge current. Second, a magnification effect of the persistent spin current is shown when one tunes the Fermi energy near the Fano-type antiresonances of the total transmission coefficient governed by the tunnel barrier strength. As an unambiguous signature of spin-orbit coupling we also show the possibility to produce a persistent *pure* spin current at the interference zeros of the transmittance. This widens the possibilities of employing mesoscopic conducting rings in phase-coherent spintronics applications.

Recently enormous attention, from both experimental and theoretical physics communities, has been devoted towards the interplay of spin-orbit (SO) coupling and quantum interference effects in confined semiconductor heterostructures. Such interplay can be exploited as a mean to control and manipulate the spin degree of freedom at mesoscopic scale useful for phase-coherent spintronic applications.[1,2] The major goal in this field is the generation of spin-polarized currents and their appropriate manipulation in a controllable environment. Since the original proposal of the spin field effect transistor (spin FET) by Datta and Das[3], many proposals have appeared based on intrinsic spin splitting properties of semiconductors associated with the Rashba spin orbit(SO) coupling[4,5]. This is a dominant mechanism that has been proven to be a convenient mean of all-electrical control of spin polarized current through additional gate voltages.[6]. In addition, suitable means for controlling spin at mesoscopic scales are provided by quantum interference effects in coherent ring conductors under the influence of electromagnetic potentials, known as Aharonov-Bohm(AB)[7] and Aharonov-Casher(AC)[8] effect. This possibility has driven a wide interest in spin-dependent transmission properties of mesoscopic AB and AC rings that have been studied under various aspects[9,10,11,12,13,14,15,16,17]. In particular, an extended literature is devoted to the study of persistent currents, from both the experimental and theoretical point of view, in closed and open quantum rings. In this context, only few papers have appeared on the analysis of persistent currents in rings where the Rashba effect is likely to be important[18,19,20,21,22]. In the early work of Splettstoesser et al.[18] the analysis of the persistent current in a ballistic mesoscopic ring with Rashba SO coupling and in absence of current leads has been employed to extract the strength of Rashba spin splitting. Unambiguous signatures of SO have also been found in Ref.[19] where the persistent spin current has been analyzed in a quantum ring with multiple arms and in presence of current leads. Here it is found that the SO coupling can increase or decrease the total persistent currents and change its direction. In this paper we focus on the persistent spin and charge currents in a one-dimensional AB ring connected to external leads, interrupted by a tunnel barrier in the lower arm and subject to Rashba spin-orbit interaction[23]. First, we revisit the subject of the spin-splitting effect on the persistent currents as a function of the Rashba coupling strength, then we extract distinct effects due to the presence of the tunnel barrier in one of the arms which have not been considered in earlier works. In particular, we show a magnification effect of the persistent spin current when the Fermi energy is tuned near the antiresonances of the total transmission coefficient caused by the presence of the tunnel barrier. Such effect is similar to the one discussed in Ref.[24] where the magnification effect on persistent currents is discussed for an open Arhonov-Bohm double-ring structure in absence of SO coupling. Jayannavar et al.[25] have also reported magnification features of the persistent currents near the conductance antiresonances in a ring with rotational symmetry breaking due to unequal arm lengths but in absence of magnetic flux. As far as we know, magnification effects on spin-currents in presence of Rashba SO interaction have not been reported yet. An important feature is represented by the possibility of having a sizeable *pure* spin current (in correspondence of zero charge current) at the interference zeros of the transmittance. An analysis of the persistent currents as a function of the effective flux induced by the spin-orbit interaction is also presented.

The system under study is depicted in Fig.1. In one dimensional rings on semiconductor structure, an effective Rashba electric field results from the asymmetric confinement along the direction (k) perpendicular to the plane of the ring[4,5]. The Hamiltonian describing the Rashba SO coupling is the following:

$$\hat{H}_{SO} = \frac{\alpha}{\hbar}(\hat{\vec{\sigma}} \times \hat{\vec{p}})_k, \qquad (1)$$

where $\hbar/2\hat{\vec{\sigma}}$ is the spin operator expressed in terms of the Pauli spin matrices, $\hat{\vec{\sigma}} = (\sigma_i, \sigma_j, \sigma_k)$ and α is the SO coupling (SOC) associated to the effective electric field along the k direction. The total Hamiltonian of a moving

electron in presence of SOC can be found in Ref.[27]. In the case of a one-dimensional ring an additional confining potential (e.g. of harmonic type) must be added in order to force the electron wave function to be localized on the ring. When only the lowest radial mode is taken into account, the resulting effective one dimensional Hamiltonian in a dimensionless form[15,26] can be written as:

$$\hat{H} = \frac{2m^\star R^2}{\hbar^2}\hat{H}_{1D} = \left(-i\frac{\partial}{\partial\varphi} + \frac{\beta}{2}\sigma_r - \frac{\Phi_{AB}}{\phi_0}\right)^2, \tag{2}$$

where $m^\star$ is the effective mass of the carrier, $\beta = 2\alpha m^\star/\hbar^2$ is the dimensionless SOC, $\sigma_r = \cos\varphi\sigma_x + \sin\varphi\sigma_y$, and additional constants have been dropped, Φ_{AB} is the Bohm-Aharonov flux and ϕ_0 is the quantum flux $\phi_0 = hc/e$. The parameter α represents the average electric field along the k direction and is assumed to be a tunable quantity. For an InGaAs-based two-dimensional electron gas, α can be controlled by a gate voltage with typical values in the range $(0.5{-}2.0)\times 10^{-11}$eVm[28,29]. The tunnel barrier localized in the lower arm of the ring is modelled by a delta potential, $v\delta(\varphi' + \frac{\pi}{2})$, where v is the dimensionless tunnel barrier strength $v = 2m^\star R^2 V/\hbar^2$, and $\varphi' = -\varphi$. The local tunnel barrier can be experimentally realized by a quantum point contact[38] and its strength controlled by a so-called split-gate voltage. As outlined in the Appendix of Ref.[26] when v is zero, one can solve the eigenvalue problem in a straightforward manner and the energy eigenvalues are:

$$E_n^\sigma = (n - \Phi_{AC}^\sigma/2\pi - \Phi_{AB}/2\pi)^2, \tag{3}$$

where $\sigma = \pm$, Φ_{AC}^σ is the so-called Aharonov-Casher phase[8] $\Phi_{AC}^\sigma = -\pi(1-\sigma\sqrt{\beta^2+1})$ At fixed energy, the dispersion relation yields the quantum numbers $n_\lambda^\sigma(E) = \lambda\sqrt{E} + \Phi^\sigma/2\pi$, where we have introduced $\Phi^\sigma = \Phi_{AC}^\sigma + \Phi_{AB}$, and the index $\lambda = \pm$ refers to right/left movers, respectively. The unnormalized eigenvectors have the general form[15,26] $\Psi_n^\sigma(\varphi) = e^{in\varphi}\chi^\sigma(\varphi)$, where $n \in$Z is the orbital quantum number. It should be noted that the spinors $\chi^\sigma(\varphi)$ are generally not aligned with the Rashba electric field, but they form a tilt angle given by $\tan\theta = -\beta$ relative to the k direction and can expressed in terms of the eigenvectors of the Pauli matrix σ_k[26]. For our purposes, we set up a scattering problem and calculate the transmission coefficient, following the method of quantum waveguide transport on networks[30,31]. One main problem is the boundary conditions at the intersection with the external leads and at the tunnel barrier. In this case it is appropriate to apply the spin-dependent version of the Griffith boundary's condition[32]. These state that (i) the wave function must be continuous and (ii) the spin density must be conserved. The same conditions apply at the location of the tunnel barrier in the lower arm[33].

We assume that when an electron moves along the upper arm in the clockwise direction from $\varphi = 0$ (see Fig.1), it acquires a phase $\Phi^\sigma/2$ at the output intersection $\varphi = \pi$, whereas the electron acquires a phase $-\Phi^\sigma/4$ in the counterclockwise direction along the lower arm when moving from $\varphi' = 0(\pi/2)$ to $\varphi' = \pi/2(\pi)$. Therefore the total phase is Φ^σ when the electron goes through the loop. The wave functions in the upper(u) and lower(d) arm of the ring can be written as:

$$\begin{aligned}\Psi_u(\varphi) &= \sum_{\sigma=\pm,\lambda=\pm} c_{u,\sigma}^\lambda e^{in_\lambda^\sigma\varphi}\chi^\sigma(\varphi),\\ \Psi_{d\alpha}(\varphi') &= \sum_{\sigma=\pm,\lambda=\pm} c_{d\alpha,\sigma}^\lambda e^{-in_\lambda^\sigma\varphi'}\chi^\sigma(\varphi'),\end{aligned} \tag{4}$$

where the index $d\alpha = d1, d2$ denotes the wave function in the two-halves of the lower branch and $n_\lambda^\sigma = \lambda kR + \Phi^\sigma/2\pi$. The wave function of the electron incident from the left lead in the left and right electrodes can be expanded as:

$$\Psi_L(x) = \Psi_i + (r_\uparrow, r_\downarrow)^T e^{-ikx},\ \Psi_R(x) = (t_\uparrow, t_\downarrow)^T e^{ikx}, \tag{5}$$

where $x = R\varphi$, r_σ and t_σ are the spin-dependent reflection and transmission coefficient, Ψ_i is the wave function of the injected electron $\Psi_i = e^{ikx}\chi^\sigma(0)$. For an incident electron from the right lead an analogous expansion is possible. This enables us to formulate the scattering matrix equation as $\hat{o} = \hat{S}\hat{i}$, where $\hat{o}, \hat{i}$ stand for outgoing and incoming wave coefficients. By applying the boundary conditions at the junctions[33] and the conservation of the currents, we have a set of equations that can be solved with respect to the transmission coefficients $t_\sigma(kL, \Phi^\sigma, z)$, where we have used $z = v/k$ and $L = \pi R$:

$$t_\sigma(kL,\Phi^\sigma,z) = \frac{8\,\sin(\frac{kL}{2})\left(-4\,\cos(\frac{kL}{2})\,\cos(\frac{\Phi^\sigma}{2}) + z\,\sin(\frac{kL}{2})\,e^{i\frac{\Phi^\sigma}{2}}\right)}{4\,z\,\cos(\frac{kL}{2}) - 2\,(5\,i + 2\,z)\,\cos(2kL) + i\,\left(2 + 8\,\cos(\Phi^\sigma) - 2\,z\,\sin(\frac{kL}{2}) + (8\,i + 5\,z)\,\sin(2kL)\right)}. \tag{6}$$

The transmission probability (or transmittance) in the spin channel σ is given by $T_\sigma = t_\sigma^\star t_\sigma$ and is related to conductance via the well-known Landauer-Büttiker formula[34]. $T_\sigma(kL, \Phi^\sigma, z)$ is periodic in kL with period 2π and in

Φ^σ with period 2π. Therefore in the following we only consider the region $0 \leq kL \leq 2\pi$ and $0 \leq \Phi^\sigma \leq 2\pi$. A remarkable feature is that the transmittance presents both resonances and antiresonances. The antiresonances become asymmetric or of Fano-type[35] in the presence of a finite tunnel barrier strength[33]. When $z = 0$ only symmetric antiresonances are possible, while only resonances exist in the single ring without a barrier and in absence of electromagnetic fluxes.

We now examine the current flows in the AB-AC ring. As electrons carry spin besides charge, their motion gives rise to a spin current other than a charge current. The difference of charge current carried by spin-up and spin-down electrons is identified with the spin current[18], $I_s = \sum_{\sigma=\pm} \sigma I_\sigma$. In the presence of spin-orbit interaction the spin currents are not simply proportional to the charge currents. As we will show a possibility could emerge in which the spin current becomes *pure*, i.e. when the charge current, $I_c = \sum_\sigma I_\sigma$, is exactly zero. The spin currents in the upper and lower arm are generally different by various symmetry breaking. In the case under investigation both time-reversal symmetry and spin-reflection symmetry are broken via the Aharonov-Bohm and Aharonov-Casher effect, respectively, while the tunnel barrier breaks the rotational symmetry. This is responsible for the persistent charge and spin current in the ring. When the current in one arm is larger than T_σ , the current in the other arm has to be negative to conserve the total current at the junction with the external leads. One can view such a negative current as a circulating current in the loop and define it as a persistent current.[25] The probability current in the upper arm is given by $I^u_\sigma = (|c^+_{u,\sigma}|^2 - |c^-_{u,\sigma}|^2)$ and can be written as:

$$I^u_\sigma = \frac{T_\sigma}{2}\left(1 - 2\frac{\tan \pi\Phi^\sigma}{\tan(kL)}\right) - F(kL, \Phi^\sigma, z), \tag{7}$$

where $F(kL, \Phi^\sigma, z)$ is complicated function of the energy, the effective flux and the tunnel barrier. With the above definition, the persistent current with spin σ is given by $I_{p\sigma} = (T_\sigma - I^u_\sigma)$, when $I^u_\sigma > T_\sigma$ and $I_{p\sigma} = -I^u_\sigma$ when I^u_σ is negative. A similar definition holds when we consider the probability current in the lower arm I^d_σ. After having identified the wave-vectors intervals wherein either I^u_σ or I^d_σ flow in the negative direction, by their magnitudes we have calculated the persistent currents per spin σ and the persistent charge and spin currents by: $\mathcal{I}_s = \sum_\sigma \sigma I_{p\sigma}$ and $\mathcal{I}_c = \sum_\sigma I_{p\sigma}$.

The transmittance and the persistent spin and charge currents in dimensionless units (respectively of $\hbar v_F/2$ and ev_F , v_F being the Fermi velocity) are shown as a function of kL (k near k_F) in Fig.2 for $\Phi_{AB} = \pi, \beta = 1.83, z = 0.1$. A remarkable feature is that in correspondence of the Fano resonance at $kL = \pi$ the amplitude of the spin-current is magnified and remarkably the spin current becomes *pure* at the interference zeros, $kL = 0, \pi$. Let us note that the amplitude of the persistent current is proportional to the slope of the Fano resonance. Such a slope diverges when kL approaches the singular points and so does the persistent current. The Fano-type resonances are present only when the tunnel barrier strength is non-zero, as discussed above. We also notice that the persistent currents change sign when crossing the energy or the wave vector at the antiresonance. In fact such antiresonance is characterized by an asymmetric pole structure in the transmission amplitude. This behavior is similar to one observed for the persistent charge currents in an open ring with incommensurate arm lengths in absence of electromagnetic fluxes[25]. The origin instead of a pure spin current near $kL = 0, 2\pi$ stems for the interference effects at the junctions in the presence of a SO interaction that induces finite transmission probability in the spin channel opposite to the incident spin orientation[36]. The divergent feature of the current near $kL = 0, 2\pi$ stems from the first term in (7) while the current divergence near $kL = \pi$ stems for the second term in (7) and has a non-trivial dependence on the tunnel barrier. The persistent currents as a function of the Aharonov-Bohm flux are reported in Fig.3 for $\beta = 1.8, z = 0.1, kL = 2\pi$. Close to the maximum of the transmittance at $\Phi_{AB} = \pi$ a pure spin current is detected. Nearby two values of the Aharonov-Bohm flux (0.8π and 1.2π) correspond to a pure charge current. The persistent charge and spin currents are shown in Fig.4 as a function of the SO coupling for $\Phi_{AB}/2\pi = 0.49, kL = 2\pi, z = 0.5$. For the values of the parameters chosen, the spin current is magnified by varying the SO coupling strength at the points where the transmittance has maxima. Further the persistent spin current oscillates between positive and negative values as the intensity of the SO coupling increases. These features further indicate that the directions of the persistent currents depend on the intensity of the SO coupling, and that it can increase or decrease the total persistent current. These findings are in agreement with those in Ref.[19] where persistent spin currents in a multiple arms ring were discussed. As a function of the tunnel barrier strength z the persistent spin current shows a minimum without sign change. This implies that z can be varied to maximize the spin currents. This is shown in Fig.5 where the parameters are $\beta = 1.5, \Phi_{AB}/2\pi = 0.45$. Finally we have verified that finite temperature effects do not lead to cancellations of the persistent current features described above, apart from a slight renormalization of the currents magnitude up to temperatures of the order $100mK$[33]at which real devices are working. In conclusion, we have analyzed the properties of the persistent spin and charge currents in an open quantum ring subject to the Rashba spin-orbit interaction in presence of an external magnetic flux and a tunnel barrier in the lower arm. We have discussed a magnification effect of the persistent spin currents in association with the Fano resonances of the transmission coefficient, depending on the magnitude of the tunnel barrier strength z. We have also shown that persistent *pure* spin currents can arise which stem for the time-reversal symmetry breaking and the spin-reversal symmetry breaking due to the total effective flux enclosed in the ring.

Finally, we have shown that the directions of the persistent currents depend on the intensity of spin-orbit coupling and the tunnel barrier strength that can increase or decrease the persistent currents. The different dependencies of the persistent charge and spin currents are a *unique* signature of the spin-orbit coupling affecting the electronic structure of the ring that can be exploited in experiments. Indeed, the possibility to measure spin persistent currents in open rings is within reach with today's technology for experiments in semiconductor heterostructures, e.g. InGaAs-based 2DEG[28,37]. Indeed spin interference effects in Rashba-gate-controlled ring with a quantum point contact inserted have recently been reported[38] and could be further investigated to reproduced the spin current magnification effects discussed here.

Acknowledgments

We thank Prof. Maria Marinaro for very useful suggestions and lightening discussions.

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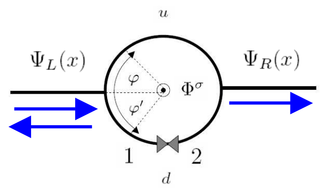

FIG. 1: One dimensional ring in presence of current leads and subject to Rashba spin-orbit interaction.

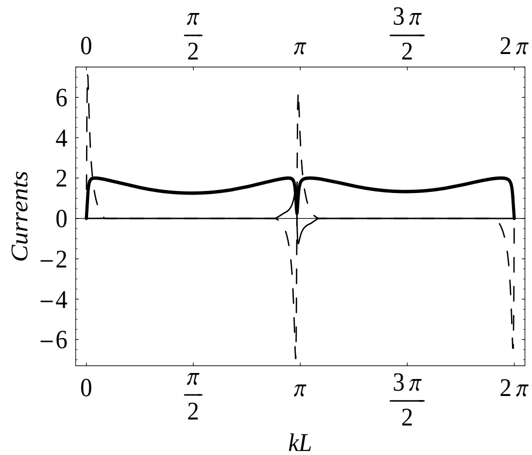

FIG. 2: Persistent currents $\mathcal{I}_c$ (thin line) and $\mathcal{I}_s$ (dashed line) in dimensionless units (of ev_F and $\hbar/2v_F$, respectively) plotted as a function of kL with $\Phi_{AB}/(2\pi) = 0.5$, $\beta = \sqrt{3}+0.1$, $z = 0.1$. The persistent currents are magnified in vicinity of a Fano-like anti-resonance in the normalized transmittance (thick line).

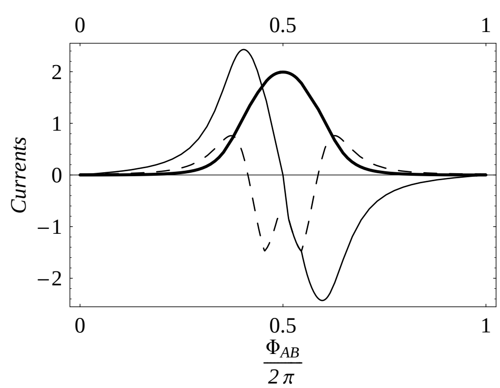

FIG. 3: Persistent currents $\mathcal{I}_c$ (thin line) and $\mathcal{I}_s$ (dashed line) in dimensionless units (see text) vs $\Phi_{AB}/(2\pi)$ with $kL = 2\pi+0.1$, $\beta = 1.8$, $z = 0.5$. A pure spin persistent currents is obtained for half-integers values of the Aharonov-Bohm flux. The thick line represents the transmittance.

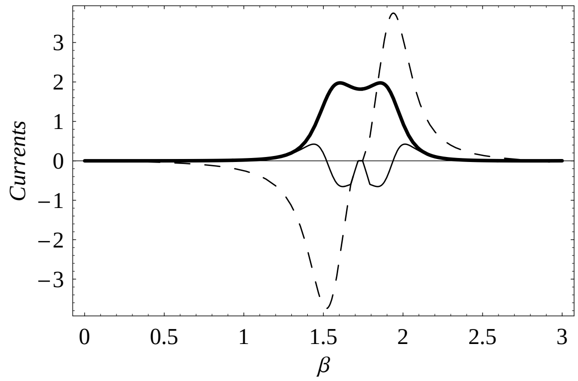

FIG. 4: Persistent currents $\mathcal{I}_c$ (thin line) and $\mathcal{I}_s$ (dashed line) in dimensionless units (see text)plotted as a function of β with $kL = 2\pi + 0.1$, $\Phi_{AB}/(2\pi) = 0.5$, $z = 0.5$. The thick line represents the transmittance.

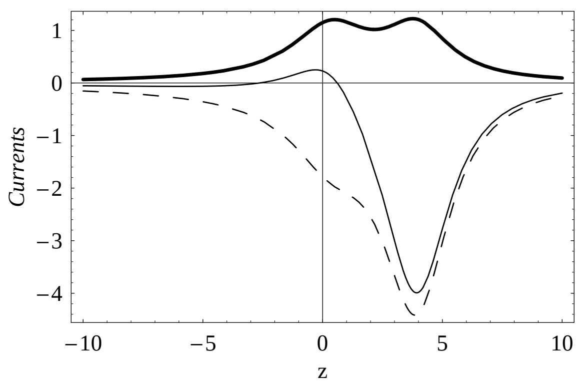

FIG. 5: Persistent currents $\mathcal{I}_c$ (thin line) and $\mathcal{I}_s$ (dashed line) in dimensionless units (see text) plotted as a function of z with $\Phi_{AB}/(2\pi) = 0.45$, $kL = 2\pi + 0.08$, $\beta = 1.5$. The normalized transmittance (thick line) is also shown.